Physics 201 Lab: Static Equilibrium and the Superposition of Forces

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Theoretical Discussion

Whenever a *net* force acts on an object, the object accelerates. Yet despite the fact that many of the objects around us in everyday life; our televisions, stereos, computers, sofas, refrigerators, etc.; are constantly subject to a large number of forces, they are not flying around in a state of perpetual acceleration. The reason that they are not is that usually, the *net* force on them is zero. A situation in which a number of non-zero forces combine to give a net force of zero is called *static equilibrium.* The manner in which forces combine is called **vector superposition**. The superposition principle for forces can be summarized as follows:

The net force that results from a number of individual forces acting on an object is the vector sum of the individual forces.

In mathematical terms, if N different forces act on an object, the net force on the object is equal to

$$\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 + \cdots + \vec{F}_N$$

Note that this is a vector equation, so that one must add the force vectors, and not merely the magnitudes of the forces. Therefore, the direction of the forces is very important. Both string forces and contact forces ultimately derive from electrostatic interactions, but the manner in which forces act through them is different. We can summarize these differences as follows:

- **Tension forces:** Tension forces, such as the forces mediated by a string, acts along the straight line defined by the direction of the string at the point where it is in contact with the object. Since a string or cord is not rigid, one can only pull, not push, with a string. Therefore the direction of the force is always away from the object. Tension force directions are shown in Figure 1A.
- **Contact forces:** The force acting between two solid bodies in contact acts at every point of contact between them, and *only* at points of contact between them. The direction of the force is along the line connecting the two points of contact.[1] For every point of contact, each body exerts an equal and opposite force on the other. The nature of contact forces are depicted in Figure 1B.

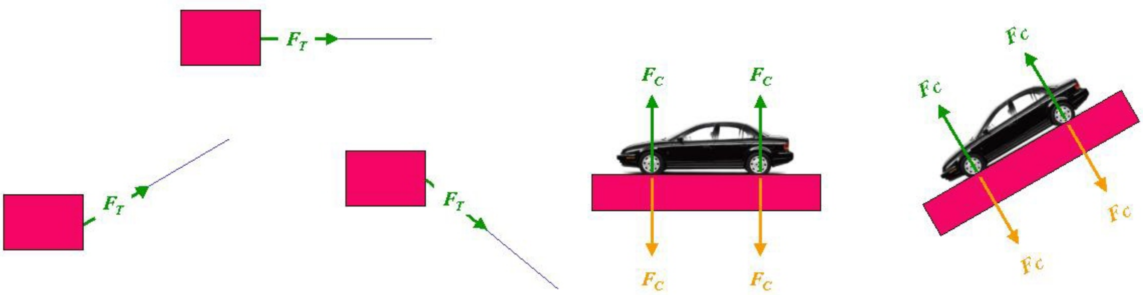

Figure 1A Figure 1B

FIG. 1: The directions of tension and contact forces.

Since this experiment entails the use of pulleys, it is important to understand the manner in which pulleys mediate forces. A pulley acts only to change the direction of a tension force. A simple pulley does not change the magnitude of the force. The action of pulleys is illustrated in Figure 2.

If the system is in static equilibrium, then the net force is zero, so that

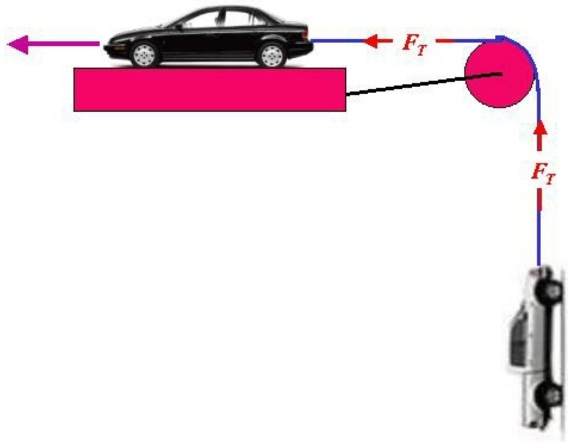

FIG. 2: Action of a simple pulley to change the direction of a tension force.

$$\vec{F}_1 + \vec{F}_2 + \cdots + \vec{F}_N = 0$$

In two dimensions, the vector equation above can be written in terms of components to give the two equations

$$\begin{aligned} (\vec{F}_1)_x + (\vec{F}_2)_x + \cdots + (\vec{F}_N)_x &= 0 \\ (\vec{F}_1)_y + (\vec{F}_2)_y + \cdots + (\vec{F}_N)_y &= 0 \end{aligned}$$

Both of these equations must be satisfied for the system to be in static equilibrium.

Procedure

The experiment is designed to confirm the principle of superposition as it applies to forces and static equilibrium. We use the force table to balance vector components of the gravitational forces due to three different masses, denoted m_1, m_2, and m_2. An overhead schematic diagram of the force table is shown in Figure 3A; Figure 3B shows a side view of the force table.

If the masses are arranged as in Figure 3A, then balancing the x and y components of the forces requires the following two equations to be satisfied.

$$m_1 g \cos\theta_1 + m_2 g \cos\theta_2 - m_3 g = 0 \tag{1}$$

$$m_1 g \sin\theta_1 - m_2 g \sin\theta_2 = 0 \tag{2}$$

To simplify the analysis, we introduce an unconventional force unit, the *gram-weight.*. One gram-weight is the weight of one gram at our laboratory site.[2] Note that adopting the gram-weight as our force unit implies that mg is numerically equal to m (in grams), making g effectively 1 in the above equations.

The procedure can be summarized as follows:

1. Choose three 'weights' stamped 150 g, 200 g, and 130 g; together with their supporting hangers, these are the masses m_1, m_2, and m_3, respectively. Use the mass balance to obtain accurate mass values – including the hanger – for all three. Report these values (in grams), to the number of significant figures justified by the measurement.

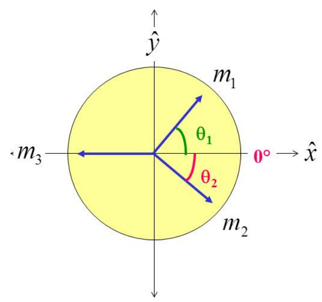

Figure 3A

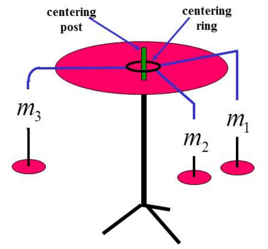

Figure 3B

FIG. 3: Overhead and side views of the force table. In gram-weight units, the force exerted by each hanging mass has magnitude equal to the mass in grams.

2. Place the mass m_3 at $\theta_B = 180°$. Adjust the angles of θ_1 and θ_2, so that the centering post is aligned in the middle of the centering ring. When this condition is achieved, the system is in static equilibrium, as shown in Figure 3A. Record the values of θ_1 and θ_2.

3. Using your measured values for θ_1, θ_2, m_1, m_2 and m_3, evaluate the left sides of Equations 1 and 2; these are the components of the net force $\vec{F}_{net}$ as determined analytically from your experiment. Report your result for $\vec{F}_{net}$ in the form of a magnitude (in gram-weights) and an angle. Because the predicted value is zero, any nonzero value for $\vec{F}_{net}$ reflects experimental error, and the magnitude of this vector becomes a reasonable indicator of the standard error in the measurement. Report this magnitude as a fraction of the magnitude of $\vec{F}_3$, the smallest of the three forces that were added.

4. Add the individual forces $\vec{F}_1$ thru $\vec{F}_3$ graphically (tail to tip construction) to obtain the net force. Report the magnitude (in gram-weights) and direction of $\vec{F}_{net}$ obtained using this method. Which method – graphical or analytical – do you think is superior, and why?

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[1] More precisely, this is the *normal component* of the contact force. The *friction component* –if any– is that part of the contact force that acts perpendicular to the connecting line, i.e., parallel to the surfaces in contact.

[2] From this definition and the accepted value for g at our location, we calculate the SI equivalent of one gram-weight to be $mg = (0.001\ \text{kg})(9.797\ \text{kg-m/s}^2) = 9.797 \times 10^{-3}$ N.